

Anomalies and Hawking radiation from the Reissner-Nordström black hole with a global monopole

Shuang-Qing Wu

College of Physical Science and Technology, Central China Normal University,
Wuhan, Hubei 430079, People's Republic of China

E-mail: sqwu@phy.ccnu.edu.cn

Jun-Jin Peng

College of Physical Science and Technology, Central China Normal University,
Wuhan, Hubei 430079, People's Republic of China

Abstract. We extend the work by S. Iso, H. Umetsu and F. Wilczek [Phys. Rev. Lett. 96 (2006) 151302] to derive the Hawking flux via gauge and gravitational anomalies of a most general two-dimensional non-extremal black hole space-time with the determinant of its diagonal metric differing from the unity ($\sqrt{-g} \neq 1$) and use it to investigate Hawking radiation from the Reissner-Nordström black hole with a global monopole by requiring the cancellation of anomalies at the horizon. It is shown that the compensating energy momentum and gauge fluxes required to cancel gravitational and gauge anomalies at the horizon are precisely equivalent to the $(1+1)$ -dimensional thermal fluxes associated with Hawking radiation emanating from the horizon at the Hawking temperature. These fluxes are universally determined by the value of anomalies at the horizon.

Keywords: Anomaly, Hawking radiation, Black hole, Global monopole

Submitted to: *Class. Quantum Grav.*

PACS numbers: 04.70.Dy, 03.65.Sq, 04.62.+v

1. Introduction

Since Hawking's remarkable discovery [1] in 1974 that a black hole is not completely black, but can emit all species of fundamental particles from its event horizon, the study of black hole radiation has been in the spotlight all the time. Hawking radiation is a kind of quantum effect that originates from the vacuum fluctuation near the horizon. This effect is a common character of almost all kinds of horizons, whose occurrence is only dependent upon the existence of a horizon. Since Hawking effect interplays the theory of general relativity with quantum field theory and statistical thermodynamics, it is generally believed that a deeper understanding of Hawking radiation may shed some lights on seeking a complete theory of quantum gravity. There are several ways to derive Hawking radiation. The original one presented by Hawking [1] is very complicated, making it not only difficult to join a concrete physical picture with calculations but also nontrivial to extend the result to other gravitational backgrounds. On the other hand, the conformal (trace) anomaly method [2], which is closely related to the recent hot spots on the subject, has several limits to be generalized to the cases of higher dimensional black holes.

Recently, Robinson and Wilczek [3] proposed an intriguing approach to derive Hawking radiation from a Schwarzschild-type black hole through gravitational anomaly. Their basic idea goes as follows. Consider a massless scalar field in the higher dimensional space-time. Upon performing a dimensional reduction technique together with a partial wave decomposition, they found that the physics near the horizon in the original black hole background can be described by an infinite collection of massless fields in a $(1+1)$ -dimensional effective field theory. If omitting the classically irrelevant ingoing modes in the region near the horizon, the effective theory becomes chiral and exhibits gravitational anomalies in the near-horizon region, which can be cancelled by the $(1+1)$ -dimensional black body radiation at the Hawking temperature. Subsequently, the case of a charged black hole was immediately studied in [4] to enclose gauge anomaly in addition to gravitational anomaly, where the authors showed that both gauge and gravitational anomalies at the horizon can be exactly cancelled by the compensating charged current flux and energy momentum flux. These fluxes are universally determined only by the value of anomalies at the horizon and are equivalent to the $(1+1)$ -dimensional thermal fluxes associated with Hawking radiation emanating from the horizon at the Hawking temperature. As is shown later, the anomaly cancellation method is very universal, and soon it has been successfully extended to other black hole cases [5, 6, 7, 8, 9].

In this paper, we will further extend this method derive the Hawking flux via gauge and gravitational anomalies of a most general two-dimensional non-extremal black hole space-time with the determinant of its diagonal metric different from the unity ($\sqrt{-g} \neq 1$) and then apply it to investigate Hawking radiation of a charged, static spherically symmetric (Reissner-Nordström) black hole with a global monopole from the viewpoint of anomaly cancellation. An unusual and stirring property of the black-hole-global-monopole system [10] is that it possesses a solid deficit angle, which

makes it quite different topologically from that of a Schwarzschild black hole and its charged counterpart alone. Because the background space-time considered here is not asymptotically flat, rather it contains a topological defect due to the presence of a global monopole, the original anomaly cancellation method, however, will become problematic to be directly applied for the black-hole–global-monopole system. To avoid this obstacle, as did in [9] we shall adopt a slightly different procedure and perform various coordinate transformations before we can take use of this method to derive Hawking radiation via the cancellation of anomalies and the regularity requirement of fluxes at the horizon.

The plan of this paper is as follows. We begin with our discussions in Sec. 2 by studying the thermodynamics of charged black holes with a global monopole. Gauge and gravitational anomalies are then analyzed in Sec. 3 for a most general non-extremal metric in two dimensions. It should be pointed out that our analysis presented this section is far more general, so it is suitable for studying Hawking radiation of wider classes of non-extremal black hole space-times where we have only assumed that $f(r) = h(r) = 0$ at the horizon, and previous researches are included as special cases of the metric considered here. Sec. 4 is devoted to applying the anomaly recipe to our charged black hole with a global monopole. As a comparison, we also include here the method of Damour-Ruffini-Sannan's (DRS) [11] tortoise coordinate transformation to tackle with the Hawking evaporation. The last section ends up with some remarks on further applications of the general analysis completed in this paper, and on the choice of the time-like Killing vector, especially in the space-time with a cosmological constant.

2. Thermodynamics of charged black holes with a global monopole

The metric of a general non-extremal Reissner-Nordström black hole with the global $O(3)$ monopole is described by [10]

$$ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

$$A = \frac{q}{r}dt, \quad f(r) = h(r) = 1 - \eta^2 - \frac{2m}{r} + \frac{q^2}{r^2}, \quad (2)$$

where m is the mass parameter of the black hole and η is related to the symmetry breaking scale when the global monopole is formed during the early universe soon after the Big Bang [12]. For a typical GUT symmetry breaking scale, $\eta^2 \sim 10^{-6}$, so it's reasonable to regard $1 - \eta^2 \simeq 1$ on physical grounds.

Since the prime physical quantity obtained by means of the anomaly cancellation method is the Hawking temperature which enters into the first law of black hole thermodynamics, let's begin with by studying the thermodynamical properties of the black-hole–global-monopole system [9, 13]. The Hawking temperature of a black hole is given in terms of its surface gravity. Note that the metric of Eq. (1) is no longer asymptotically flat, so the well known formula

$$\kappa = \frac{1}{2} \sqrt{\frac{g_{rr}}{-g_{tt}}} (-g_{tt, r})|_{r=r_+}, \quad (3)$$

for computing the surface gravity for a general spherically symmetric asymptotically flat metric, becomes potentially problematic to be applied in the case described by the metric (1). It is worth noting that the metric (1) is still asymptotically bounded. In order to get the Hawking temperature of the black hole with a global monopole, we first derive a general formula for calculating the surface gravity for asymptotically bounded spherically symmetric black hole space-times, and then apply it to the case considered here.

A spherically symmetric asymptotically bounded space-time metric can, without loss of generality, be cast in the form

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (4)$$

with the metric coefficients $g_{tt} = -f(r)$ and $g_{rr} = 1/h(r)$ approximating two finite constants at the infinity. The surface gravity for the black hole described by Eq. (4) can be found from the relation

$$\kappa^2 = -\frac{1}{2}l_{\mu;\nu}l^{\mu;\nu}|_{r=r_+}, \quad (5)$$

where $l^\mu = C^{-1}\delta_t^\mu$ is the time-like Killing vector, in which one can take the constant $C = 1$ or $C = \lim_{r \rightarrow \infty} \sqrt{-g_{tt}} = \sqrt{1 - \eta^2}$, according to whether the normalized condition $\lim_{r \rightarrow \infty} l_\mu l^\mu = -1$ is adopted or not (when $C = 1$, $\lim_{r \rightarrow \infty} l_\mu l^\mu = \eta^2 - 1$). After some calculation, we then find

$$\kappa^2 = \frac{g^{rr}}{4C^2(-g_{tt})}(-g_{tt,r})^2|_{r=r_+} = \frac{h}{4C^2 f} f_{,r}^2|_{r=r_+}, \quad (6)$$

and so the surface gravity is

$$\kappa = \frac{1}{2C} \sqrt{\frac{g^{rr}}{-g_{tt}}}(-g_{tt,r})|_{r=r_+} = \frac{1}{2C} \sqrt{\frac{h}{f}} f_{,r}|_{r=r_+}. \quad (7)$$

This is the general formula to calculate the surface gravity for spherically symmetric asymptotically bounded black hole space-times and it reduces to Eq. (3) for the asymptotically flat case where we have $C = 1$.

For the space-time metric (1), the surface gravity at the horizon for the combined black-hole-global-monopole system is explicitly given by

$$\kappa = \frac{1}{2C} \sqrt{f_{,r} h_{,r}}|_{r=r_+} = \frac{(1 - \eta^2)r_+ - m}{Cr_+^2}. \quad (8)$$

The Arnowitt-Deser-Misner mass M of the system can be calculated via the Komar integral

$$M = \frac{1}{8\pi} \oint l_{(t)}^{\mu;\nu} d^2\Sigma_{\mu\nu} = \frac{m}{C}, \quad (9)$$

where we have used $l_{(t)}^\mu = C^{-1}(\partial_t)^\mu$. Obviously, the mass M isn't equal to the mass parameter m when $C \neq 1$ because of the presence of a global monopole. The electric charge is determined by

$$Q = \frac{1}{4\pi} \oint F^{\mu\nu} d^2\Sigma_{\mu\nu} = q, \quad (10)$$

and the electric potential is

$$\Phi = l^\mu A_\mu|_{r=r_+} = \frac{q}{Cr_+}. \quad (11)$$

Finally, the area of the horizon can be computed via

$$A = \int_{r=r_+} \sqrt{g_{\theta\theta}g_{\varphi\varphi}} d\theta d\varphi = 4\pi r_+^2. \quad (12)$$

One can easily show that the mass M , the charge Q , the Hawking temperature $T = \kappa/(2\pi)$, the entropy $S = A/4$, and the electric potential Φ given above obey the differential and integral forms of the first law of black hole thermodynamics as follows

$$dM = TdS + \Phi dQ, \quad M = 2TS + \Phi Q. \quad (13)$$

Now introducing the following coordinate transformation

$$t = (1 - \eta^2)^{-1/2} \tilde{t}, \quad r = (1 - \eta^2)^{1/2} \tilde{r}, \quad (14)$$

and defining two new parameters

$$m = (1 - \eta^2)^{3/2} \tilde{m}, \quad q = (1 - \eta^2) \tilde{q}, \quad (15)$$

then we can rewrite the line element (1) as

$$ds^2 = -\tilde{f}(\tilde{r}) d\tilde{t}^2 + \tilde{h}(\tilde{r})^{-1} d\tilde{r}^2 + (1 - \eta^2) \tilde{r}^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (16)$$

$$A = \frac{\tilde{q}}{\tilde{r}} d\tilde{t}, \quad \tilde{f}(\tilde{r}) = \tilde{h}(\tilde{r}) = 1 - \frac{2\tilde{m}}{\tilde{r}} + \frac{\tilde{q}^2}{\tilde{r}^2}. \quad (17)$$

This metric is, apart from the deficit solid angle $4\pi\eta^2$, very similar to the Reissner-Nordström solution.

Thermodynamical quantities can be analogously computed. In the following, we will take $C = \sqrt{1 - \eta^2}$ for the line element (1) in order to be consistent with the above coordinate transformation. This means that we shall use the Killing vector $l^\mu = (1 - \eta^2)^{-1/2} (\partial_t)^\mu = (\partial_{\tilde{t}})^\mu$ to compute the mass M , the surface gravity κ , and the electric potential Φ for the line elements (1) and (16). They are given by

$$M = \frac{m}{\sqrt{1 - \eta^2}} = (1 - \eta^2) \tilde{m}, \quad (18)$$

$$\kappa = \frac{(1 - \eta^2) r_+ - m}{\sqrt{1 - \eta^2} r_+^2} = \frac{\tilde{r}_+ - \tilde{m}}{\tilde{r}_+^2}, \quad (19)$$

$$\Phi = \frac{q}{\sqrt{1 - \eta^2} r_+} = \frac{\tilde{q}}{\tilde{r}_+}. \quad (20)$$

The horizon area and the electric charge can be similarly evaluated as

$$A = 4\pi r_+^2 = 4\pi (1 - \eta^2) \tilde{r}_+^2, \quad (21)$$

$$Q = q = (1 - \eta^2) \tilde{q}, \quad (22)$$

where $r_+ = (1 - \eta^2)^{-1} [m + \sqrt{m^2 - (1 - \eta^2) q^2}]$ and $\tilde{r}_+ = \tilde{m} + \sqrt{\tilde{m}^2 - \tilde{q}^2}$. It is easily to check that they satisfy the Bekenstein-Smarr's relationship (13).

It should be emphasized that the space-time (1) is not asymptotically flat due to the presence of a global monopole; while the metric (16) is 'quasi'-asymptotically flat

if regardless of the deficit solid angle. In addition, there is also one another slight difference in the expression of the surface gravity. The surface gravity of the metric (1) is $\kappa = \frac{1}{2\sqrt{1-\eta^2}}f_{,r}(r_+)$, which corresponds to the normalized time-like Killing vector $l_{(t)}^\mu = (1 - \eta^2)^{-1/2}(\partial_t)^\mu$ (when $C = \sqrt{1 - \eta^2}$ is adopted); while for the metric (16), it has the familiar form $\kappa = \frac{1}{2}\tilde{f}_{,\tilde{r}}(\tilde{r}_+)$ with respect to the time-like Killing vector $\partial_{\tilde{t}}$. This means that in each case one must use the normalized time-like Killing vector to obtain the correct physical quantities. If, however, one prefers to choose ∂_t for the time-like Killing vector for the space-time metric (1) where $C = 1$, then the temperature, the mass, and the electric potential differ from the ones given in Eqs. (18-20) by a multiplier $\sqrt{1 - \eta^2}$. For this reason, the original anomaly cancellation method can be immediately used to obtain the consistent formula of Hawking temperature for the metric (16), but not directly for the line element (1). Thus, we shall first base our analysis upon the metric (16) but will soon turn to the space-time (1).

3. Anomalies of a most general charged black hole in two dimensions

In this section, we will further extend the anomaly cancellation method [4] to show that the fluxes of Hawking radiation from a most general spherically symmetric non-extremal black hole can be determined by anomaly cancellation conditions and regularity requirements at the horizon.

The reason why we need to make such a generalization is due to the following aspects. First of all, the case of $\sqrt{-g} = 1$ is clearly too limited, and almost all of the previously related studies [3, 4, 5, 6, 7, 8] are limited to this case. [Nevertheless, there is a freedom to choose an appropriate dilaton factor to rescale the desired metric so as to meet with the assertion $\sqrt{-g} = 1$.] Secondly, a most general diagonal metric with $\sqrt{-g} \neq 1$ is more natural, and this is especially needed for studies of dilatonic black holes in string theory and Klein-Kaluza theory (see, for example, the first two references in [7]). Thus it is physically important to do such an extension.

3.1. Dimensional reduction

The metric and gauge potential of a most general, static and spherically symmetric non-extremal black hole space-time are given by

$$ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + P(r)^2d\Omega^2, \quad (23)$$

$$A = A_t dt = \frac{q}{r} dt, \quad (24)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element on the unit 2-sphere. We assume[‡] that the functions $f(r)$ and $h(r)$ vanish at the horizon $r = r_+$ of the black hole, that is, $f(r) \rightarrow f_{,r}(r_+)(r - r_+)$ and $h(r) \rightarrow h_{,r}(r_+)(r - r_+)$ as $r \rightarrow r_+$. At the horizon $r = r_+$, the surface gravity is given by $\kappa = \frac{1}{2}\sqrt{f_{,r}h_{,r}}|_{r_+}$.

Consider now the dimensional reduction from $d = 4$ to $d = 2$. For simplicity, let's consider the action for a massless scalar field in the black hole background (23). After performing a partial wave decomposition $\phi = \sum_{lm} \phi_{lm}(t, r) Y_{lm}(\theta, \varphi)$, and only keeping the dominant terms near the horizon, the action becomes

$$\begin{aligned}
S[\phi] &= -\frac{1}{2} \int d^4x \sqrt{-g_{(4)}} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = \frac{1}{2} \int d^4x \sqrt{-g_{(4)}} \phi \square \phi \\
&= \frac{1}{2} \int dt dr d\theta d\varphi P^2 \sin\theta \sqrt{-g} \phi \left\{ -\frac{1}{f} \partial_t^2 + h \partial_r^2 \right. \\
&\quad \left. + \left[\frac{(fh)_{,r}}{2f} + \frac{2h}{P} P_{,r} \right] \partial_r + \frac{1}{P^2} \Delta_\Omega \right\} \phi \\
&= \frac{1}{2} \sum_{lm} \int dt dr P^2 \sqrt{-g} \phi_{lm} \left\{ -\frac{1}{f} \partial_t^2 + h \partial_r^2 \right. \\
&\quad \left. + \left[\frac{(fh)_{,r}}{2f} + \frac{2h}{P} P_{,r} \right] \partial_r - \frac{l(l+1)}{P^2} \right\} \phi_{lm} \\
&\simeq \frac{1}{2} \sum_{lm} \int dt dr P^2 \sqrt{-g} \phi_{lm} \left[-\frac{1}{f} \partial_t^2 + h \partial_r^2 + \frac{(fh)_{,r}}{2f} \partial_r \right] \phi_{lm}, \quad (25)
\end{aligned}$$

where Δ_Ω is the angular Laplace operator, here and hereafter $\sqrt{-g} = \sqrt{f/h}$.

Upon transforming to the tortoise coordinate defined by $r_* = \int dr / \sqrt{fh}$, one finds that the effective radial potentials for partial wave modes of the field vanish exponentially fast near the horizon. Thus the physics near the horizon can be described by an infinite collection of massless fields in the $(1+1)$ -dimensional effective theory, each partial wave propagating in a space-time with a metric given by the “ $r - t$ ” section of the full space-time metric (23) and the dilaton field $\Psi = P(r)^2$, which makes no contribution to the anomaly.

The same procedure applies to the case of a massive charged scalar field with a mass term and a minimal electro-magnetic coupling interaction in a background space-time (23). Similarly, we can get the $(1+1)$ -dimensional effective metric and the gauge potential

$$ds^2 = -f(r) dt^2 + h(r)^{-1} dr^2, \quad (26)$$

$$A_t = \frac{q}{r}, \quad (27)$$

[‡] At the first sight, the second one of this assumption is stronger than the previous one ($f(r_+) = h(r_+) = 0$) and seems to exclude the extremal black hole. Nevertheless, the analysis done in this section is still applicable to the extremal black hole case. The result we obtained is a zero Hawking temperature, which means that there is no Hawking radiation, so there is no anomaly needed to be cancelled at the horizon. This is in accordance with the fact that an extremal black hole is supersymmetric. For this reason, we shall concentrate ourself to the non-extremal case only.

with the dilaton field $\Psi = P(r)^2$. It should be pointed out that we have implicitly used the regular behavior of the dilaton field at the horizon in the above dimensional reduction process.

Apparently, a scalar field in the original $(3 + 1)$ -dimensional background can be effectively described by an infinite collection of massless fields in the $(1 + 1)$ -dimensional background space-time with the effective metric and the gauge potential given by Eq. (26), together with the dilaton field $\Psi = P(r)^2$. Now apply the above analysis to the metric (16), we will arrive at (26) with a dilaton field $\Psi = (1 - \eta^2)r^2$. On the other hand, if we start with the metric (1), the same effective metric yields but with a different dilaton factor $\Psi = r^2$. In each case, $f(r) = h(r)$ finds its corresponding expression given in Eq. (16) or (1).

At this stage, it should be noted that when reducing to $d = 2$, the Lagrangian contains a factor $\Psi = P(r)^2$, which can be interpreted as a dilaton background coupled to the charged fields [14]. Since we are considering a static background, the contribution from the dilaton field can be neglected. In addition, the effective two dimensional current is given by integrating the 4-dimensional ones over a 2-sphere, $J^\mu = \int \sqrt{-g} P^2 \sin \theta d\theta d\varphi J_{(4)}^\mu = 4\pi \sqrt{-g} P^2 J_{(4)}^\mu(r)$. For later use, we include here the non-vanishing Christoffel symbols and the Ricci scalar for the effective metric (26)

$$\Gamma_{tr}^t = \Gamma_{rt}^t = \frac{f_{,r}}{2f}, \quad \Gamma_{tt}^r = \frac{hf_{,r}}{2}, \quad \Gamma_{rr}^r = -\frac{h_{,r}}{2h}, \quad (28)$$

$$R = -\frac{hf_{,rr}}{f} - \frac{f_{,r}h_{,r}}{2f} + \frac{hf_{,r}^2}{2f^2}. \quad (29)$$

3.2. Gauge anomaly

As is well known, Hawking effect takes place in the region near the horizon. Since the horizon is a one-way membrane, modes interior to the horizon can not affect physics outside the horizon, classically. Therefore one can only consider the physics outside the horizon, and define the effective theory in the outer region $[r_+, +\infty]$. The exterior region can be divided into two parts: the near-horizon region $[r_+, r_+ + \varepsilon]$, and the other region $[r_+ + \varepsilon, +\infty]$. In the latter region, which is far away from the horizon, the theory is not chiral, the current of energy momentum tensor and the charged current are conserved. On the contrary, in the near-horizon region where there are only outgoing modes, if we formally neglect quantum effects of the ingoing modes since such modes never come out once they fall into black holes, the effective theory becomes chiral there and contains gauge and gravitational anomalies associated with gauge invariance and diffeomorphism invariance, respectively. But because the underlying theory is invariant under gauge and diffeomorphism symmetries, these anomalies must be cancelled by quantum effects of the modes that are classically irrelevant. In what follows, we reveal that the conditions for cancellation of these anomalies at the horizon are, in turn, met by the Hawking flux of charge and energy momentum, with the Hawking temperature exactly being that for the considered space-time.

First we investigate the charged current and gauge anomaly at the horizon. We localize the physics outside the horizon since the effective theory is defined in the exterior region $[r_+, +\infty]$, and focus on the gauge anomaly in the region $[r_+, r_+ + \varepsilon]$. If we first omit the classically irrelevant ingoing modes in the near-horizon region $[r_+, r_+ + \varepsilon]$, the gauge current exhibits an anomaly there. The consistent form of $d = 2$ Abelian anomaly is given by [15]

$$\nabla_\mu J^\mu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu, \quad (30)$$

while the covariant current is defined by [15]

$$\nabla_\mu \tilde{J}^\mu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}F_{\mu\nu}, \quad (31)$$

where a minus sign $(-)$ corresponds to right-handed fields and $\epsilon^{\mu\nu}$ is an antisymmetric tensor with notation $\epsilon^{tr} = 1$. The consistent anomaly satisfies the Wess-Zumino condition but the current J^μ transforms non-covariantly. The coefficient of the covariant anomaly is *twice* as large as that of the consistent anomaly. The covariant current is related to the consistent one by

$$\tilde{J}^\mu = J^\mu + \frac{e^2}{4\pi\sqrt{-g}}A_\lambda\epsilon^{\lambda\mu}. \quad (32)$$

For the non-vanishing component of the charged currents, we have

$$\tilde{J}^r = J^r + \frac{e^2}{4\pi\sqrt{-g}}A_t(r)H(r). \quad (33)$$

In the other region $[r_+ + \varepsilon, +\infty]$, where the theory is not chiral and there is no anomaly, the current $J_{(O)}^\mu$ is conserved. On the other hand, in the near-horizon region $[r_+, r_+ + \varepsilon]$, since there are only outgoing (right handed) fields, the effective quantum field theory is chiral and exhibits a gauge anomaly with respect to gauge symmetry, the current $J_{(H)}^\mu$ satisfies the anomalous equation. Obviously, these currents must obey, respectively, the following equations

$$\nabla_\mu J_{(O)}^\mu = 0, \quad (34)$$

$$\nabla_\mu J_{(H)}^\mu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}\partial_\mu A_\nu. \quad (35)$$

Hence we can write them out explicitly with respect to the metric ansatz (26)

$$\partial_r[\sqrt{-g}J_{(O)}^r] = 0, \quad (36)$$

$$\partial_r[\sqrt{-g}J_{(H)}^r] = \frac{e^2}{4\pi}\partial_r A_t, \quad (37)$$

and solve them in each region as

$$\sqrt{-g}J_{(O)}^r = c_O, \quad (38)$$

$$\sqrt{-g}J_{(H)}^r = c_H + \frac{e^2}{4\pi}[A_t(r) - A_t(r_+)], \quad (39)$$

where c_O and c_H are integration constants.

The total current outside the horizon is written as a sum of two regions $J^\mu = J_{(O)}^\mu \Theta(r) + J_{(H)}^\mu H(r)$, where $\Theta(r) = \Theta(r - r_+ - \epsilon)$ is a scalar step function, $H(r) = 1 - \Theta(r)$ is a scalar top hat function. By using the anomaly equation, the variation of the effective action (without the omitted ingoing modes near the horizon) under gauge transformations becomes

$$\begin{aligned} -\delta_\lambda W &= \int dt dr \sqrt{-g} \lambda \nabla_\mu J^\mu \\ &= \int dt dr \lambda \left\{ \partial_r \left(\frac{e^2}{4\pi} A_t H \right) + \left[\frac{e^2}{4\pi} A_t \right. \right. \\ &\quad \left. \left. + \sqrt{-g} (J_{(O)}^r - J_{(H)}^r) \right] \delta(r - r_+ - \epsilon) \right\}, \end{aligned} \quad (40)$$

where λ is a gauge parameter.

The total effective action must be gauge invariant and the first term should be cancelled by quantum effects of the classically irrelevant ingoing modes. The quantum effect to cancel this term is the Wess-Zumino term induced by the ingoing modes near the horizon. In order to keep the gauge invariance, the variation of the effective action should vanish, $\delta_\lambda W = 0$. The coefficient of the delta-function should also vanish at the horizon, namely

$$\sqrt{-g} [J_{(O)}^r - J_{(H)}^r](r_+) + \frac{e^2}{4\pi} A_t(r_+) = 0, \quad (41)$$

which relates the coefficient of the current in two regions:

$$c_O = c_H - \frac{e^2}{4\pi} A_t(r_+), \quad (42)$$

where c_H is the value of the consistent current at the horizon.

In order to determine the current flow, we need to fix the value of the current at the horizon. Since the condition should be gauge covariant, we impose that the coefficient of the *covariant* current at the horizon should vanish. Since $\tilde{J}_{(H)}^r = J_{(H)}^r + e^2 A_t(r)/(4\pi\sqrt{-g})$, the condition $\tilde{J}_{(H)}^r = 0$ determines the value of the charge flux to be

$$\begin{aligned} c_H &= -\frac{e^2}{4\pi} A_t(r_+), \\ c_O &= -\frac{e^2}{2\pi} A_t(r_+) = -\frac{e^2 q}{2\pi r_+}. \end{aligned} \quad (43)$$

This agrees with the current flow associated with the Hawking thermal (blackbody) radiation including a chemical potential, as will appear presently.

3.3. Gravitational anomaly

Next we turn to discuss the gravitational anomaly and the flow of the energy momentum tensor. A gravitational anomaly is an anomaly in the general coordinate covariance, taking the form of non-conservation of energy momentum tensor. If we neglect quantum effects of the ingoing modes near the horizon, the energy momentum tensor

in the effective theory exhibits a gravitational anomaly with respect to diffeomorphism invariance in the region $[r_+, r_+ + \varepsilon]$. The consistent anomaly arising in the $(1+1)$ -dimensional chiral theory reads [15]

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} \equiv \mathcal{A}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_\nu, \quad (44)$$

for right-handed fields. The covariant anomaly in $1+1$ dimensions, on the other hand, takes the form [15]

$$\nabla_\mu \tilde{T}^\mu_\nu = \frac{-1}{96\pi\sqrt{-g}}\epsilon_{\mu\nu}\partial^\mu R \equiv \tilde{\mathcal{A}}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \quad (45)$$

Since we are considering a static background, the contribution to the anomaly from the dilaton background can be dropped. Let's first ignore the electro-magnetic interaction and only concentrate on the pure gravitational anomaly. In the far-away-horizon region $[r_+ + \varepsilon, +\infty]$, the energy momentum tensor $T^\mu_{(O)\nu}$ is conserved; but in the near-horizon region $[r_+, r_+ + \varepsilon]$, the energy momentum tensor $T^\mu_{(H)\nu}$ obey the anomalous equation. Obviously these equations are

$$\nabla_\mu T^\mu_{(O)\nu} = 0, \quad (46)$$

$$\nabla_\mu T^\mu_{(H)\nu} \equiv \mathcal{A}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_\nu. \quad (47)$$

For a metric of the form (26), $N^\mu_\nu = \mathcal{A}_\nu = 0$ in the region $[r_+ + \varepsilon, +\infty]$. But in the near-horizon region $[r_+, r_+ + \varepsilon]$, the components of N^μ_ν are

$$N^r_t = \frac{1}{96\pi}\partial_r\Gamma^r_{tt} = \frac{1}{192\pi}(f_{,r}h_{,r} + hf_{,rr}), \quad N^r_r = 0, \quad (48)$$

$$N^t_r = \frac{-1}{96\pi}\partial_r\Gamma^r_{rr} = \frac{-1}{192\pi h^2}(h_{,r}^2 - hh_{,rr}), \quad N^t_t = 0. \quad (49)$$

The consistent and covariant anomalies are purely time-like ($\mathcal{A}_r = \tilde{\mathcal{A}}_r = 0$), and can be written as

$$\begin{aligned} \sqrt{-g}\mathcal{A}_t &= \partial_r N^r_t = \partial_r[\sqrt{-g}T^r_t] = \frac{1}{96\pi}\partial_r^2\Gamma^r_{tt} \\ &= \frac{1}{192\pi}\partial_r(hf_{,rr} + f_{,r}h_{,r}), \end{aligned} \quad (50)$$

$$\begin{aligned} \sqrt{-g}\tilde{\mathcal{A}}_t &= \partial_r \tilde{N}^r_t = \partial_r[\sqrt{-g}\tilde{T}^r_t] = \frac{-1}{96\pi}f\partial_r R \\ &= \frac{1}{96\pi}\partial_r\left(hf_{,rr} + \frac{f_{,r}h_{,r}}{2} - \frac{hf_{,r}^2}{f}\right). \end{aligned} \quad (51)$$

Making use of $N^r_t = (hf_{,rr} + h_{,r}f_{,r})/(192\pi)$ and $\tilde{N}^r_t = [hf_{,rr} + f_{,r}h_{,r}/2 - hf_{,r}^2/f]/(96\pi)$, we relate the covariant energy momentum tensor to the consistent one by

$$\sqrt{-g}\tilde{T}^r_t = \sqrt{-g}T^r_t + \frac{h}{192\pi f}\left(ff_{,rr} - 2f_{,r}^2\right). \quad (52)$$

We now include the electro-magnetic interaction. Since the complex scalar field is in a fixed background of the electric field and dilaton field, the energy momentum tensor is not conserved even classically. We first derive the appropriate Ward identity.

Under diffeomorphism transformations $\delta x^\mu = -\xi^\mu$, metric and gauge field transform as $\delta g^{\mu\nu} = -(\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu)$ and $\delta A_\mu = \xi^\nu \partial_\nu A_\mu + A_\nu \partial_\mu \xi^\nu$. Since the action for matter fields $S[g_{\mu\nu}, A_\mu]$ should be invariant (we neglect the contribution of the dilaton field), hence, if there were no gravitational anomaly, the Ward identity becomes

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu} J^\mu + A_\nu \nabla_\mu J^\mu. \quad (53)$$

Here we have kept the term proportional to the gauge anomaly. Adding the gravitational anomaly, the Ward identity becomes

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu} J^\mu + A_\nu \nabla_\mu J^\mu + \mathcal{A}_\nu = F_{\mu\nu} \tilde{J}^\mu + \mathcal{A}_\nu. \quad (54)$$

In the other region $[r_+ + \varepsilon, +\infty]$, the energy momentum tensor $T_{(O)\nu}^\mu$ and the charged current $J_{(O)}^\mu$ are conserved; but in the near-horizon region $[r_+, r_+ + \varepsilon]$, the charged current $J_{(H)}^\mu$ obeys the anomaly equation, and the energy momentum tensor $T_{(H)\nu}^\mu$ satisfies a modified anomalous equation in each region, the energy momentum tensors must be subjected to

$$\nabla_\mu T_{(O)\nu}^\mu = F_{\mu\nu} J_{(O)}^\mu, \quad (55)$$

$$\nabla_\mu T_{(H)\nu}^\mu = F_{\mu\nu} J_{(H)}^\mu + A_\nu \nabla_\mu J_{(H)}^\mu + \mathcal{A}_\nu = F_{\mu\nu} \tilde{J}_{(H)}^\mu + \mathcal{A}_\nu. \quad (56)$$

We now solve the above identities for the $\nu = t$ component. In the exterior region without anomalies, the identity is

$$\partial_r [\sqrt{-g} T_{(O)t}^r] = \sqrt{-g} F_{rt} J_{(O)}^r = c_O \partial_r A_t, \quad (57)$$

where we have utilized $F_{rt} = \partial_r A_t$ and $T_t^r = -f h T_r^t$. By using $\sqrt{-g} J_{(O)}^r = c_O$, it is solved as

$$\sqrt{-g} T_{(O)t}^r = a_O + c_O A_t(r) = a_O - \frac{e^2}{2\pi} A_t(r_+) A_t(r), \quad (58)$$

where a_O is an integration constant. Since there is a gauge and gravitational anomaly near the horizon, the Ward identity becomes

$$\begin{aligned} \partial_r [\sqrt{-g} T_{(H)t}^r] &= \sqrt{-g} [F_{rt} J_{(H)}^r + A_t \nabla_\mu J_{(H)}^\mu] + \partial_r N_t^r \\ &= \sqrt{-g} F_{rt} \tilde{J}_{(H)}^r + \partial_r N_t^r, \end{aligned} \quad (59)$$

where the first and the second term has been combined to become $\sqrt{-g} F_{rt} \tilde{J}_{(H)}^r$. By substituting $\sqrt{-g} \tilde{J}_{(H)}^r = c_O + e^2 A_t(r)/(2\pi)$ into this equation, we can solve $T_{(H)t}^r$ via

$$\begin{aligned} \partial_r [\sqrt{-g} T_{(H)t}^r] &= \sqrt{-g} J_{(H)}^r \partial_r A_t + A_t \partial_r [\sqrt{-g} J_{(H)}^r] + \partial_r N_t^r \\ &= \sqrt{-g} \tilde{J}_{(H)}^r \partial_r A_t + \partial_r N_t^r, \end{aligned} \quad (60)$$

explicitly as

$$\sqrt{-g} T_{(H)t}^r = a_H + \left(c_O A_t + \frac{e^2}{4\pi} A_t^2 + N_t^r \right) \Big|_{r_+}^r. \quad (61)$$

The total energy momentum tensor outside the horizon combines contributions from these two regions: $T_\nu^\mu = T_{(O)\nu}^\mu \Theta(r) + T_{(H)\nu}^\mu H(r)$, in which $T_{(O)\nu}^\mu$ is covariantly conserved

and $T_{(H)\nu}^\mu$ obeys the anomalous Eq. (47). Under the infinitesimal general coordinate transformations, the effective action changes as

$$\begin{aligned} -\delta_\xi W &= \int dt dr \sqrt{-g} \xi^\nu \nabla_\mu T^\mu_\nu \\ &= \int dt dr \xi^t \left\{ c_O \partial_r A_t + \partial_r \left[\left(\frac{e^2}{4\pi} A_t^2 + N_t^r \right) H \right] \right. \\ &\quad \left. + \left[\sqrt{-g} (T_{(O)t}^r - T_{(H)t}^r) + N_t^r + \frac{e^2}{4\pi} A_t^2 \right] \delta(r - r_+ - \epsilon) \right\} \\ &\quad + \int dt dr \xi^r \sqrt{-g} (T_{(O)r}^r - T_{(H)r}^r) \delta(r - r_+ - \epsilon). \end{aligned} \quad (62)$$

The first term is the classical effect of the background electric field for constant current flow. The second term should be cancelled by the quantum effect of the ingoing modes. In order to restore the diffeomorphism invariance, the variation of the effective action should vanish. The coefficient of the last term should also vanish at the horizon. Setting $\delta_\xi W = 0$, we obtain the following constraints

$$\sqrt{-g} [T_{(O)t}^r - T_{(H)t}^r](r_+) + N_t^r(r_+) + \frac{e^2}{4\pi} A_t^2(r_+) = 0, \quad (63)$$

which relates the coefficients:

$$a_O = a_H + \frac{e^2}{4\pi} A_t^2(r_+) - N_t^r(r_+) = \frac{e^2}{4\pi} A_t^2(r_+) + N_t^r(r_+). \quad (64)$$

In order to determine a_O , we need to fix the value of the energy momentum tensor at the horizon. As before, we impose a vanishing condition for the *covariant* energy momentum tensor at the horizon, $\tilde{T}_{(H)t}^r = 0$. This additional regularity condition leads to

$$a_H = 2N_t^r(r_+) = \frac{f_{,r} h_{,r}}{96\pi} \Big|_{r=r_+} = \frac{\kappa^2}{24\pi}, \quad (65)$$

where κ is the surface gravity of the black hole. The total flux of the energy momentum tensor is given by

$$a_O = \frac{e^2 q^2}{4\pi r_+^2} + N_t^r(r_+) = \frac{e^2 q^2}{4\pi r_+^2} + \frac{\kappa^2}{48\pi}. \quad (66)$$

3.4. Blackbody radiation

In the uncharged case [9], we have shown that $\sqrt{-g} T_{(O)t}^r = N_t^r(r_+)$ is the energy momentum flux of Hawking radiation. A $(1+1)$ -dimensional black body radiation at temperature T has a flux of the form $N_t^r(r_+) = (\pi/12)T^2$, accurately giving the Hawking temperature $T = \kappa/(2\pi)$.

In the present charged case, we now compare the results (43) and (66) with the fluxes from blackbody radiation at a temperature $T = \kappa/(2\pi)$ with a chemical potential $\omega_0 = eA_t(r_+) = eq/r_+$. The Planck distribution in a charged black hole is given by

$$I^{(\pm)}(\omega) = \frac{1}{e^{2\pi(\omega \pm \omega_0)/\kappa} - 1}, \quad J^{(\pm)}(\omega) = \frac{1}{e^{2\pi(\omega \pm \omega_0)/\kappa} + 1}, \quad (67)$$

for bosons and fermions, respectively. $I^{(-)}$ and $J^{(-)}$ correspond to the distributions for particles with charge $-e$. To keep things simple, we only consider the fermion case. With these distributions, the fluxes of charged current and energy momentum become

$$\sqrt{-g}J^r = \int_0^\infty e \frac{d\omega}{2\pi} [J^{(-)}(\omega) - J^{(+)}(\omega)] = -\frac{e^2 q}{2\pi r_+}, \quad (68)$$

$$\sqrt{-g}T_t^r = \int_0^\infty \omega \frac{d\omega}{2\pi} [J^{(-)}(\omega) + J^{(+)}(\omega)] = \frac{e^2 q^2}{4\pi r_+^2} + \frac{\kappa^2}{48\pi}. \quad (69)$$

The results (43) and (66) derived from the anomaly cancellation conditions coincide with these results (68) and (69), showing that the required thermal flux is capable of cancelling the anomaly. Thus the compensating energy momentum flux and charged current flux required to cancel gravitational and gauge anomalies at the horizon are precisely equivalent to thermal fluxes associated with a $(1+1)$ -dimensional blackbody radiation emanating from the horizon at the Hawking temperature.

4. Hawking radiation of charged black holes with global monopoles

In this section, we will apply the above analysis to consider Hawking radiation from a Reissner-Nordström black hole with the global monopole. As is shown in [9], because the considered space-time is not asymptotically flat, the original anomaly cancellation method cannot be immediately applied to obtain the consistent formula of Hawking temperature for the line element (1), rather it can be directly used to obtain that for the metric (16). Thus, we shall first base our analysis below upon the metric (16) but will soon turn to the space-time (1). By comparison, we also include here the DRS method to deal with the Hawking evaporation of a most general two-dimensional non-extremal metric (26).

4.1. Hawking radiation: the anomaly recipe

The general formula for the Hawking temperature derived via the anomaly cancellation method is

$$T = \frac{\kappa}{2\pi} = \frac{\sqrt{f_{,r}h_{,r}}}{4\pi} \Big|_{r=r_+}. \quad (70)$$

Specialize to the present case where $f(r) = h(r)$, we get the Hawking temperature $T = f_{,r}(r_+)/ (4\pi)$. Apply this formula to the metric (16), we can obtain the consistent temperature $T = \tilde{f}_{,\tilde{r}}(\tilde{r}_+)/ (4\pi) = f_{,r}(r_+)/ (4\pi\sqrt{1-\eta^2})$, which corresponds to the time-like Killing vector $\partial_{\tilde{t}}$ or the normalized time-like Killing vector $(1-\eta^2)^{-1/2}\partial_t$ (where $C = \sqrt{1-\eta^2}$). On the other hand, if we straight-forwardly apply it to the line element (1), we will get a different result $T = f_{,r}(r_+)/ (4\pi)$ with respect to the Killing vector ∂_t (when $C = 1$ is adopted). So it is unadvisable to apply directly the original anomaly cancellation method to the space-time (1), otherwise one must divide the pure energy momentum flux by a factor $1-\eta^2$. Nevertheless, we can do the same analysis in another

different way. By re-scaling $t \rightarrow \sqrt{1 - \eta^2} t$, we rewrite the metric (1) as

$$ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + r^2d\Omega^2, \quad (71)$$

$$f(r) = \frac{h(r)}{1 - \eta^2}, \quad h(r) = 1 - \eta^2 - \frac{2m}{r} - \frac{q^2}{r^2}, \quad (72)$$

and immediately derive the expected result for the Hawking temperature $T = f_{,r}(r_+)/ (4\pi\sqrt{1 - \eta^2})$.

The analysis for the electric potential proceeds in the similar manner and will not be repeated here.

4.2. Hawking radiation: the DRS method

As mentioned before, a major physical quantity derived by the anomaly cancellation method is the Hawking temperature of the black hole. The issue of this method is that it is closely related to the near-horizon conformal property of the black hole geometry. Since it has already been proved that the Hawking temperature is conformal invariant [16], so the surface gravity can also simply derived by the conformal transformation method. This goes as follows.

Introducing the tortoise coordinate r_* defined by $r_* = \int dr/\sqrt{fh}$, the generic two-dimensional metric (26) can be written in a conformal form

$$ds^2 = f(r)(-dt^2 + dr_*^2). \quad (73)$$

Near the horizon $r \rightarrow r_+$, the radial coordinate r has the asymptotic behavior

$$r_* \approx \frac{1}{2\kappa} \ln(r - r_+), \quad (74)$$

where $\kappa = \frac{1}{2}\sqrt{f_{,r}h_{,r}}|_{r=r_+}$ can be further shown to be the surface gravity of the black hole.

Another well-known method to examine the Hawking radiation is the DRS method, which is also intimately related to the conformal property of the black hole. We now elucidate it as follows. Without loss of generality, we now consider a complex scalar field Ψ with the mass μ_0 and the charge e . The charged scalar field equation with a minimal electro-magnetic interaction in the background space-time (26) is given by

$$\square_c \Psi = \left[-\frac{1}{f}(\partial_t + ieA_t)^2 + \sqrt{\frac{h}{f}}\partial_r(\sqrt{fh}\partial_r) - \mu_0^2 \right] \Psi = 0. \quad (75)$$

Separation of variables as $\Psi(t, r) = R(r)e^{-i\omega t}$ yields the radial equation

$$[\partial_{r_*}^2 + (\omega - eA_t)^2 - \mu_0^2 f]R(r) = 0. \quad (76)$$

At the infinity, the radial part becomes the free wave equation. Near the horizon $r \rightarrow r_+$, it has a standard form of the wave equation

$$[\partial_{r_*}^2 + (\omega - \omega_0)^2]R(r_*) = 0, \quad (77)$$

where $\omega_0 = eA_t(r_+) = eq/r_+$ is the Coulomb energy.

The ingoing wave solution and the outgoing wave solution are, respectively,

$$R_{\text{in}} = e^{-i\omega t - i(\omega - \omega_0)r_*}, \quad (78)$$

$$R_{\text{out}} = e^{-i\omega t + i(\omega - \omega_0)r_*} = R_{\text{in}} e^{2i(\omega - \omega_0)r_*}. \quad (79)$$

The ingoing wave solution R_{in} is analytic at the horizon, but the outgoing wave solution

$$R_{\text{out}} \approx R_{\text{in}}(r - r_+)^{i(\omega - \omega_0)/\kappa}, \quad (r > r_+), \quad (80)$$

has a logarithmic singularity at the horizon $r = r_+$, it can be analytically continued from the outside of the hole into the inside of the hole along the lower complex r -plane

$$(r - r_+) \rightarrow (r_+ - r)e^{-i\pi} \quad (81)$$

to

$$\widetilde{R}_{\text{out}} = R_{\text{in}}(r_+ - r)^{i(\omega - \omega_0)/\kappa} e^{\pi(\omega - \omega_0)/\kappa}, \quad (r < r_+). \quad (82)$$

The relative scattering probability at the event horizon is

$$\left| \frac{R_{\text{out}}}{\widetilde{R}_{\text{out}}} \right|^2 = e^{-2\pi(\omega - \omega_0)/\kappa}. \quad (83)$$

Following the DRS method [11], the bosonic spectrum of Hawking radiation of scalar particles from the black hole is easily obtained

$$\langle \mathcal{N}(\omega) \rangle = \frac{1}{e^{2\pi(\omega - \omega_0)/\kappa} - 1}. \quad (84)$$

Similarly, the fermionic spectrum of Hawking radiation of Dirac particles from the black hole can also be deduced

$$\langle \mathcal{N}(\omega) \rangle = \frac{1}{e^{2\pi(\omega - \omega_0)/\kappa} + 1}. \quad (85)$$

From the radiant spectra, the Hawking temperature can be determined as $T = \kappa/(2\pi)$.

Finally, we briefly mention that the DRS method has been further developed in Ref. [17] to a so-called generalized tortoise coordinate transformation (GTCT) method and subsequently it has been shown to be a powerful tool to investigate Hawking radiation of a larger class of non-stationary black holes. This GTCT method can simultaneously determine not only the location of the event horizon, but also the Hawking temperature as well as the thermal radiant spectrum of any kinds of black holes, whether they are static, stationary or non-stationary.

5. Concluding remarks

In this paper, we have further extended the work in Ref. [4] to derive the Hawking flux from a most general two-dimensional non-extremal black hole space-time with the determinant of its diagonal metric differing from the unity and applied it to investigate Hawking radiation of a Reissner-Nordström black hole with the global monopole by requiring the cancellation of anomalies at the horizon. The regularity condition that requires the covariant charged current flux and the covariant energy momentum flux to vanish at the horizon is corresponding to the choice of the Unruh vacuum state. The

conditions for anomaly cancellation at the horizon are met by the Hawking flux of energy momentum and charged current flux. These fluxes have the forms precisely equivalent to black body radiation with the Hawking temperature. To obtain a consistent expression of the Hawking temperature, it is not suitable to directly use as a starting point the space-time metric (1) which is not asymptotically flat due to the presence of a global monopole.

It should be pointed out that our general analysis presented here is suitable for studying Hawking radiation of wider classes of non-extremal black hole space-times where we have only assumed that $f(r_+) = h(r_+) = 0$, thus previous researches are included as special cases of the metric considered here. Especially, it can be directly applied to the case of a Reissner-Nordström-anti-de Sitter black hole with a global monopole where $f(r) = h(r) = 1 - \eta^2 - 2m/r + q^2/r^2 + r^2/l^2$, except a modification in the calculation [18] of the Arnowitt-Deser-Misner mass of the black hole.

A special point on the choice of the time-like Killing vector to calculate the surface gravity should be emphasized here a little more. The space-time considered in this paper is not asymptotically flat but asymptotically bounded due to the presence of a global monopole. In general, one prefers to choose the normalized time-like Killing vector $l^\mu = (1 - \eta^2)^{-1} \delta_t^\mu$ so that the normalized condition $\lim_{r \rightarrow \infty} l_\mu l^\mu = -1$ is satisfied. However, this kind of normalized choice will fail in the case when the black hole has a nonzero cosmological constant. Alternatively, if one chooses $l^\mu = \delta_t^\mu$ as the time-like Killing vector ($\lim_{r \rightarrow \infty} l_\mu l^\mu = -1 + \eta^2$), then he obtains a different result for the surface gravity. Thus a different choice of the time-like Killing vector will lead to a different expression for the Hawking temperature, which may differ by a multiplier $\sqrt{1 - \eta^2}$ in the cases considered in this paper.

Acknowledgements

S.-Q.Wu was supported in part by the National Natural Science Foundation of China under Grant No. 10675051 and by a starting fund from Central China Normal University.

References

- [1] Hawking S 1974 *Nature* (London) **248** 30-31
Hawking S 1975 *Commun. Math. Phys.* **43** 199-220
- [2] Christensen S M and Fulling S A 1977 *Phys. Rev. D* **15** 2088-2104
Balbinot R, Fabbri A and Shapiro I 1999 *Phys. Rev. Lett.* **83** 1494-1497 (*Preprint* hep-th/9904074)
Balbinot R and Fabbri A 1999 *Phys. Rev. D* **59** 044031 (*Preprint* hep-th/9807123)
Kummer W and Vassilevich D V 1999 *Phys. Rev. D* **60** 084021 (*Preprint* hep-th/9811092)
Hofmann and Kummer W 2005 *Eur. Phys. J. C* **40** 275-286 (*Preprint* gr-qc/0408088)
- [3] Robinson S P and Wilczek F 2005 *Phys. Rev. Lett.* **95** 011303 (*Preprint* gr-qc/0502074)
- [4] Iso S, Umetsu H and Wilczek F 2006 *Phys. Rev. Lett.* **96** 151302 (*Preprint* hep-th/0602146)
- [5] Iso S, Umetsu H and Wilczek F 2006 *Phys. Rev. D* **74** 044017 (*Preprint* hep-th/0606018)
Murata K and Soda J 2006 *Phys. Rev. D* **74** 044018 (*Preprint* hep-th/0606069)

- Iso S, Morita H and Umetsu H 2007 *J. High. Energy Phys.* **JHEP04(2007)068** (*Preprint* hep-th/0612286)
- [6] Vagenas E C and Das S 2006 *J. High. Energy Phys.* **JHEP10(2006)025** (*Preprint* hep-th/0606077)
- Setare M R 2007 *Euro. Phys. J. C* **49** 865-868 (*Preprint* hep-th/0608080)
- Xu Z B and Chen B 2007 *Phys. Rev. D* **75** 024041 (*Preprint* hep-th/0612261)
- Xiao K, Liu W B and Zhang H B 2007 *Phys. Lett. B* **647** 482-485 (*Preprint* hep-th/0702199)
- [7] Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Rev. D* **75** 064029 (*Preprint* hep-th/0701235)
- Wu S Q 2007 *Phys. Rev. D* **76** 029904 (E)
- Jiang Q Q and Wu S Q 2007 *Phys. Lett. B* **647** 200-206 (*Preprint* hep-th/0701002)
- Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Lett. B* **651** (2007) 58-64 (*Preprint* hep-th/0701048)
- Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Lett. B* **651** (2007) 65-70 (*Preprint* arXiv:0705.3871 [hep-th])
- [8] Iso S, Morita T and Umetsu H 2007 *Phys. Rev. D* **75** 124004 (*Preprint* hep-th/0701272)
- Iso S, Morita T and Umetsu H 2007 *Phys. Rev. D* **76** (in press) *Preprint* arXiv:0705.3494 [hep-th]
- Shin H and Kim W 2007 *J. High. Energy Phys.* **JHEP06(2007)012** (*Preprint* arXiv:0705.0265 [hep-th])
- Kim W and Shin H 2007 *J. High. Energy Phys.* **JHEP07(2007)070** (*Preprint* arXiv:0706.3563 [hep-th])
- Jiang Q Q 2007 *Class. Quantum Grav.* **24** (in press) *Preprint* arXiv:0705.2068 [hep-th]
- Das S, Robinson S P and Vagenas E C 2007 *Preprint* arXiv:0705.2233 [hep-th]
- Chen B and He W 2007 *Preprint* arXiv:0705.2984 [gr-qc]
- Miyamoto U and Murata K 2007 *Preprint* arXiv:0705.3150 [hep-th]
- Murata K and Miyamoto U 2007 *Preprint* arXiv:0707.0168 [hep-th]
- Banerjee R and Kulkarni S 2007 *Preprint* arXiv:0707.2449 [hep-th]
- [9] Peng J J and Wu S Q 2007 *Chin. Phys.* (in press) *Preprint* arXiv:0705.1225 [hep-th]
- [10] Barriola M and Vilenkin A 1989 *Phys. Rev. Lett.* **63** 341-343
- [11] Damour T and Ruffini R 1976 *Phys. Rev. D* **14** 332-334
- Sannan S 1988 *Gen. Relativ. Gravit.* **20** 239-246
- [12] Preskill J P 1979 *Phys. Rev. Lett.* **43** 1365-1368
- [13] Yu H W 1994 *Nucl. Phys. B* **430** 427-440
- Jing J L, Yu H W and Wang Y J 1993 *Phys. Lett. A* **178** 59-61
- [14] Mukhanov V, Wipf A and Zelnikov A 1994 *Phys. Lett. B* **332** 283-291 (*Preprint* hep-th/9403018)
- Kummer W and Vassilevich D V 1999 *Ann. Phys. (Berlin)* **8** 801-827 (*Preprint* gr-qc/9907041)
- [15] Alvarez-Gaume L and Witten E 1984 *Nucl. Phys. B* **234** 269-330
- Bardeen W A and Zumino B 1984 *Nucl. Phys. B* **244** 421-453
- Bertlmann R and Kohlprath E 2001 *Ann. Phys. (N.Y.)* **288** 137-163 (*Preprint* hep-th/0011067)
- [16] Jacobson T and Kang G 1993 *Class. Quantum Grav.* **10** L201-L206
- [17] Zhao Z 1992 *Chin. Phys. Lett.* **9** 501-504
- Wu S Q and Cai X 2001 *Chin. Phys. Lett.* **18** 485-487 (*Preprint* gr-qc/0102081)
- [18] Dutta S and Gopakumar R 2006 *Phys. Rev. D* **74** 044007 (*Preprint* hep-th/0604070)